Optimal Growth Policy: the Role of Skill Heterogeneity

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Abstract

A simple semi-endogenous growth model is employed to show that optimal subsidization of both R&D and capital costs is independent of the distribution of R&D skills in the workforce. This holds despite the empirically supported fact that a higher R&D subsidy rate raises wages of R&D workers.

Key words: Optimal growth policy; R&D skills; R&D subsidy; Semi-endogenous growth, Heterogeneity.
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1 Introduction

Empirical evidence suggests that the social return to R&D significantly exceeds the private return (e.g. Grilichis and Lichtenberg, 1984; Jones and Williams, 1998). Likewise, calibrated R&D-based growth models find a substantial R&D underinvestment gap (e.g. Jones and Williams, 2000). These results seem to call for large R&D subsidies (Grossmann, Steger and Trimborn, 2010a,b).

However, a frequent critique of raising R&D subsidies is that R&D skills are in limited supply and those with high R&D skills may already be allocated to R&D occupations. It has been argued that, therefore, stimulating demand for R&D workers primarily raises wages of R&D personnel rather than fostering innovation. In fact, Goolsbee (1998) shows that remuneration of R&D workers is positively affected by public policies to support R&D. It therefore seems that the distribution of R&D skills affects the optimal R&D policy: more limited R&D skills should imply lower optimal R&D subsidies.

We introduce R&D skill heterogeneity in a standard semi-endogenous growth model à la Jones (1995) and show analytically that, surprisingly, the optimal R&D policy is in fact *independent* of the distribution of R&D skills. The wedge between R&D investment in market equilibrium and social planning optimum is solely driven by positive R&D externalities. This holds despite a positive relationship between R&D subsidies and wages of R&D workers. Moreover, the optimal capital subsidy is also independent of the skill distribution.

2 The Model

Time is continuous and indexed by t. The time index is omitted whenever this does not lead to confusion. Homogenous final output is produced according to

$$Y = (L^Y)^{1-\alpha} \int_0^A x(i)^\alpha \mathrm{d}i, \tag{1}$$

$0 < \alpha < 1$, where L^Y denotes labor in manufacturing, A the "number" of intermediate goods and $x(i)$ the quantity of intermediate good i. The final goods sector is perfectly competitive and the output price is normalized to unity.

There is perfect competition in the R&D sector. The number of ideas evolves according to

$$\dot{A} = \nu A^\phi L^A, \tag{2}$$

where L^A is R&D labor input (in efficiency units), $\nu > 0$, $\phi < 1$. The price mark-up charged by each firm cannot exceed $\kappa \in (1, 1/\alpha]$, due to the existence of a competitive fringe (e.g. Aghion and Howitt, 2005). Parameter κ captures the degree of imperfection of goods market competition. The capital stock, $K = \int_0^A x(i) \mathrm{d}i$, depreciates at rate $\delta \geq 0$. Both the capital and labor market are perfect. Initial levels of state variables, K_0 and A_0, are given.

There is mass one of identical households indexed by $j \in [0, 1]$. The size of each household grows with constant exponential rate, $n \geq 0$. Normalizing $N_0 = 1$, household size and population size at time t are both equal to $N_t = e^{nt}$. In contrast to standard R&D-based growth models, we allow for R&D skill heterogeneity. Each household j inelastically supplies either one unit of labor to the final goods sector or $h(j)$ efficiency units to the R&D sector. R&D skills are the same within each household. The probability density function of h is denoted by $f(h)$, which is continuous with support $[\underline{h}, \bar{h}]$, $0 \leq \underline{h} < \bar{h}$. In equilibrium, the workers with the highest R&D skills are allocated to the R&D sector. That is, there will be a threshold skill level $\tilde{h}$ at which workers are indifferent in which sector to work. R&D workers with $h > \tilde{h}$ will earn a wage premium compared to workers in final good production. The amount of efficiency units of R&D labor is given by $L^A = N \int_{\tilde{h}}^{\bar{h}} h f(h) \mathrm{d}h$. Consequently, the ratio of R&D workers in efficiency units to the size of the overall workforce $l^A \equiv \frac{L^A}{N}$ is

$$l^A = \int_{\tilde{h}}^{\bar{h}} h f(h) \mathrm{d}h. \tag{3}$$

The labor resource constraint reads $L^Y + N^A = N$, where $N^A \equiv N \int_{\tilde{h}}^{\bar{h}} f(h) \mathrm{d}h$ is the number of workers allocated to R&D.

The government may subsidize both R&D costs (R&D sector) and capital costs (intermediate goods sector), at time-invariant rates s_A and s_K. Subsidies are financed by a lump-sum tax (T) on households and the government budget is balanced each period.

Preferences of household $j \in [0,1]$ are given by

$$U(j) = \int_0^\infty \frac{(c_t(j))^{1-\sigma} - 1}{1 - \sigma} e^{-(\rho - n)t} \mathrm{d}t, \tag{4}$$

$\sigma > 0$, where $c(j)$ is consumption per member of household j. Households take factor prices as given. Let w^A and w^Y denote wage rates per efficiency unit of R&D labor and for a worker in final production. Moreover, let $I(j)$ be the wage income of a member of household j. We will have

$$I(j) = \begin{cases} w^A h(j) \text{ if } h(j) \geq \tilde{h}, \\ w^Y \text{ otherwise.} \end{cases} \tag{5}$$

Financial wealth of individual j, $a(j)$, accumulates according to

$$\dot{a}(j) = (r - n)a(j) + I(j) - c(j) - T, \tag{6}$$

where r denotes the interest rate and $a_0(j) > 0$. For the transversality conditions to hold and the value of utility streams to be finite, we impose

$$\rho - n + (\sigma - 1)g > 0 \text{ with } g \equiv \frac{n}{1 - \phi}. \tag{A1}$$

3   Analysis

We first analyze the decentralized equilibrium and then the social planner solution. This allows us to derive the optimal growth policy mix.

3.1 Market Equilibrium

As one unit of capital is required for one unit of output and capital costs are subsidized according to s_K, producer i has profits

$$\pi(i) = [p(i) - (1 - s_K)(r + \delta)] x(i), \tag{7}$$

where $p(i)$ is the price of good $x(i)$. Given that the mark-up is constrained by $\kappa \in (1, 1/\alpha]$, the optimal supply price of each firm i reads (see Grossmann et al., 2010b)

$$p(i) = \kappa(1 - s_K)(r + \delta). \tag{8}$$

According to (1), the inverse demand function for intermediate good i is given by $p(i) = \alpha(L^Y/x(i))^{1-\alpha}$. Combining this inverse demand function with (8) and solving for $x(i)$ we obtain

$$x(i) = x = \left(\frac{\alpha}{\kappa(1 - s_K)(r + \delta)}\right)^{\frac{1}{1-\alpha}} L^Y. \tag{9}$$

Using (9) in (7), we see that $\pi(i) = \pi \; \forall \; i$. Using (1) for aggregate income and $K = Ax$ for the capital stock, (9) also implies

$$Y = A \left(\frac{\alpha}{\kappa(1 - s_K)(r + \delta)}\right)^{\frac{\alpha}{1-\alpha}} L^Y, \tag{10}$$

$$\frac{K}{Y} = \frac{\alpha}{\kappa(1 - s_K)(r + \delta)}. \tag{11}$$

For a given interest rate (which is policy-independent in the long run), an increase in s_K raises capital-output ratio K/Y, whereas a higher mark-up (κ) reduces it.

Let P^A denote the value of an intermediate good firm. The usual capital market equilibrium condition reads

$$\frac{\dot{P}^A}{P^A} + \frac{\pi}{P^A} = r. \tag{12}$$

The representative R&D firm maximizes profits as given by

$$\Pi \equiv P^A \underbrace{\nu A^\phi L^A}_{=\dot{A}} - (1 - s_A) w^A L^A. \tag{13}$$

Definition 1. A market equilibrium consists of time paths for the quantities $\{L_t^A, L_t^Y, \{c_t(j), a_t(j)\}_{j \in [0,1]}, \{x_t(i)\}_{i \in [0, A_t]}, Y_t, K_t, A_t\}_{t=0}^\infty$, threshold skill level $\{\tilde{h}_t\}_{t=0}^\infty$, lump-sum tax $\{T_t\}_{t=0}^\infty$ and prices $\{P_t^A, \{p_t(i)\}_{i \in [0, A_t]}, w_t^A, w_t^Y, r_t\}_{t=0}^\infty$ such that final goods producers, intermediate goods producers and R&D firms maximize profits, each household j chooses the consumption path to maximize (4) s.t. (6) and supplies labor to the R&D sector if and only if $w^A h(j) \geq w^Y$ (i.e. $\tilde{h} = \frac{w^Y}{w^A}$), the capital resource constraint $\int_0^A x(i) \mathrm{d}i = K$ holds, the labor market clears,

$$l^Y \equiv \frac{L^Y}{N} = 1 - \int_{\tilde{h}}^{\bar{h}} f(h) \mathrm{d}h, \tag{14}$$

the capital market equilibrium condition (12) holds, the goods markets clear, the financial market clears (i.e. $\int_0^N a(j) \mathrm{d}j = K + P^A A$), and the lump-sum tax T balances the government budget each period.

Let us denote values of stationary variables in balanced growth equilibrium (BGE) with superscript (*). Proofs are relegated to an online-appendix.

Proposition 1. *There exists a unique BGE such that:*

(i) The number of ideas A grows at rate $g = \frac{n}{1-\phi}$. Consumption and asset levels, $c(j)$, $a(j)$, also grow at rate $g \ \forall \ j$. Aggregate final output, Y, and the capital stock, K, grow at rate $g + n$. The value of an innovation, P^A, grows at rate n.

(ii) There is a unique stationary long-run threshold skill level, $\tilde{h}^*$, which determines if a worker j is allocated to final goods production (for $h(j) < \tilde{h}$) or R&D (for $h(j) \geq \tilde{h}$); $\tilde{h}^*$ is decreasing in the R&D subsidy rate, s_A, and independent of the capital subsidy rate, s_K.

Proposition 1 suggests that subsidizing physical capital does not affect the long run

allocation of labor, whereas an increase in the R&D subsidy rate stimulates the R&D activity of firms (i.e., l^{A*} increases and l^{Y*} decreases).

An important variable in the innovation literature is the fraction of workers allocated to R&D, $d \equiv \int_{\tilde{h}}^{\bar{h}} f(h)\mathrm{d}h$.

Corollary 1. *The long run fraction of labor allocated to R&D, d^*, is (i) increasing in the R&D subsidy rate, s_A, and (ii) not systematically affected by the distribution of R&D skills.*

Part (i) is straightforward since threshold level $\tilde{h}^*$ in BGE is decreasing in s_A. The intuition for part (ii) is discussed below.

4 Social Planning Optimum and Optimal Policy

The social planner chooses a symmetric capital allocation across intermediate good production sites, i.e., $x(i) = K/A \ \forall \ i$. From (1) one then gets $Y = K^\alpha(AL^Y)^{1-\alpha}$. The aggregate capital stock evolves according to $\dot{K} = Y - Nc - \delta K$, where c denotes per capita consumption. Hence, in per capita terms we have

$$\dot{k} = k^\alpha(Al^Y)^{1-\alpha} - c - (\delta + n)k. \tag{15}$$

As preferences are homothetic, there exists a representative consumer (Mas-Colell, Whinston and Green, 1995). The social planner's problem thus reads

$$\max_{\{c_t\}} \int_0^\infty \frac{(c_t)^{1-\sigma} - 1}{1 - \sigma} e^{-(\rho-n)t}\mathrm{d}t \ \ \text{s.t. (2), (3), (14), (15),} \tag{16}$$

and non-negativity constraints. $c, \tilde{h}$ are control variables and k, A are state variables. We focus on the BGE when comparing the first best solution with the market outcome.

Proposition 2. *In the long-run social planning optimum:*

(i) The number of ideas A and per capita consumption c grow at rate g, whereas Y and K both grow at rate $g + n$.

(ii) There exists a unique socially optimal long-run threshold skill level, $\tilde{h}^{opt}$, which is stationary. For $s_A = 0$ (no R&D subsidy) and $\phi \geq 0$, we have $\tilde{h}^{opt} < \tilde{h}^*$, i.e., there is R&D underinvestment.

Proposition 3. *The socially optimal, long-run R&D and capital cost subsidy rates are*

$$s_K^{opt} = 1 - \frac{1}{\kappa}, \tag{17}$$

$$s_A^{opt} = 1 - \frac{1 - 1/\kappa}{1/\alpha - 1} \frac{(\sigma - 1)g + \rho}{\sigma g + \rho - n}, \tag{18}$$

which are independent of the distribution of R&D skills.

There are two sources of inefficiency of R&D investments. First, if $\phi > 0$, there is a standing on shoulders effect, not taken into account by R&D firms, which promotes underinvestment. Second, innovators can only appropriate part of the economic surplus from raising the knowledge stock. To see this, first note that $x(i) = \frac{K}{A} = \frac{K}{Y}\frac{Y}{A}$ $\forall \ i$. Substituting this into (7) and using (8) and (11) reveals that $\pi = \alpha(1 - \frac{1}{\kappa})\frac{Y}{A}$. According to (10), $\frac{\partial Y}{\partial A} = \frac{Y}{A}$. Since $\alpha(1 - \frac{1}{\kappa}) < 1$, the profit of an innovator π is lower than the contribution of an additional idea to output, $\frac{\partial Y}{\partial A}$. This "surplus appropriability problem" promotes underinvestment. Overall, decentralized R&D investment is suboptimally low, calling for $s_A > 0$, whenever $\phi \geq 0$.

Due to monopolistic competition, intermediate goods supply and therefore the demand for capital are inefficiently low as well. This implies suboptimally slow capital accumulation, which calls for a subsidy on capital costs.

The novel result is that both the R&D underinvestment gap and optimal growth policy are independent of the distribution of R&D skills. When raising demand for R&D workers by subsidizing R&D costs, more R&D workers enter R&D occupations. This is socially desirable although the additional R&D workers possess lower skills than the ones already active. All that matters for the R&D underinvestment gap are the market imperfections which bias the demand for R&D skills. The main assumptions which drive our "independence result" is the possibility of firms to discriminate wage payments according to skill and that all workers possess positive R&D skills in the

relevant upper end of the distribution of R&D skills. Less skilled R&D workers earn lower income than higher skilled ones. Thus, even if the additional R&D workers, which are employed after an increase in R&D subsidization, are of lower skill, the decision of firms to hire them is not affected by the skill distribution. The possibility of wage discrimination is also the reason why the distribution of skills does not systematically affect the allocation of labor (part (ii) of Corollary 1).

Our results are consistent with the finding of Goolsbee (1998) that R&D workers gain from higher R&D subsidies. To see this, note from $\tilde{h} = \frac{w^Y}{w^A}$ (Definition 1) that long-run wage income of a R&D worker j with skill $h(j) \geq \tilde{h}^*$, relative to a production worker, is equal to the ratio of his skill to the one of the marginal entrant into a R&D occupation:

$$\frac{w^A h(j)}{w^Y} = \frac{h(j)}{\tilde{h}^*}. \tag{19}$$

According to part (i) of Proposition 1, even in the long run an increase in R&D subsidy rate s^A, by reducing threshold skill level $\tilde{h}^*$, benefits those who would be R&D workers also without the subsidy. However, this does not mean that R&D subsidization is problematic. To the contrary, it is a possibility to lure additional workers into R&D occupations, which is socially desirable.

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Online-Appendix

Proof of Proposition 1: The current-value Hamiltonian which corresponds to the intertemporal optimization problem of household j is given by

$$\mathcal{H}(j) = \frac{c(j)^{1-\sigma} - 1}{1 - \sigma} + \lambda(j)\left[(r - n)a(j) + I(j) - c(j) - T\right], \tag{20}$$

where $\lambda(j)$ is the co-state-variable associated with constraint (6). Necessary optimality conditions are $\partial\mathcal{H}(j)/\partial c(j) = 0$, $\dot{\lambda}(j) = (\rho-n)\lambda(j)-\partial\mathcal{H}(j)/\partial a(j)$, and the corresponding transversality condition. Thus,

$$\lambda(j) = c(j)^{-\sigma}, \text{ i.e., } \frac{\dot{\lambda}(j)}{\lambda(j)} = -\sigma\frac{\dot{c}(j)}{c(j)}, \tag{21}$$

$$\frac{\dot{\lambda}(j)}{\lambda(j)} = \rho - r, \tag{22}$$

$$\lim_{t\to\infty} \lambda_t(j)e^{-(\rho-n)t}a_t(j) = 0. \tag{23}$$

Combining (21) with (22), we obtain the standard Euler equation

$$\frac{\dot{c}(j)}{c(j)} = \frac{r - \rho}{\sigma} \tag{24}$$

Next, substitute (8) and (9) into (7) to obtain the following expression for the profit of each intermediate goods producer i:

$$\pi(i) = \pi = (\kappa - 1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}\left[(1 - s_K)(r + \delta)\right]^{-\frac{\alpha}{1-\alpha}} L^Y. \tag{25}$$

Since final goods producers take the wage rate w^Y as given, it is equal to its marginal productivity of labor, $w^Y = (1-\alpha)Y/L^Y$. Also recall from Definition 1 that $w^Y = w^A\tilde{h}$.

Thus,

$$w^A = \frac{(1-\alpha)Y}{\tilde{h}L^Y} \qquad (26)$$

$$= \frac{(1-\alpha)A}{\tilde{h}} \left(\frac{\alpha}{\kappa(1-s_K)(r+\delta)}\right)^{\frac{\alpha}{1-\alpha}}, \qquad (27)$$

where (27) follows by using (10). Moreover, the profit-maximizing choice of the R&D sector implies $\Pi = 0$; thus, $P^A \nu A^\phi = (1-s_A)w^A$, according to (13). Substituting (26) into the latter equation implies

$$P^A = \frac{(1-s_A)(1-\alpha)}{\nu A^{\phi-1}\tilde{h}} \left(\frac{\alpha}{\kappa(1-s_K)(r+\delta)}\right)^{\frac{\alpha}{1-\alpha}}, \qquad (28)$$

We derive the steady state by assuming that part (i) of Proposition 1 holds and show that the implications of this assumption are consistent with the assumption. For later use, setting $\dot{c}(j)/c(j) = g$ in (24) implies that the long run interest rate reads

$$r^* = \sigma g + \rho. \qquad (29)$$

Substituting (25) into (12) and setting $\dot{P}^A/P^A = n$ implies that

$$n + \frac{(\kappa-1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}} L^Y}{[(1-s_K)(r+\delta)]^{\frac{\alpha}{1-\alpha}} P^A} = r. \qquad (30)$$

From (2), we find $\dot{A}/A = \nu A^{\phi-1} L^A$. From this it becomes clear that setting $\dot{A}/A = g$ is consistent with L^A growing at rate n. Thus, in steady state,

$$\nu A^{\phi-1} = \frac{g}{Nl^A}. \qquad (31)$$

Using (31) in (28) and substituting the resulting expression as well as (29) in (30) implies that

$$\frac{1-\frac{1}{\kappa}}{\frac{1}{\alpha}-1}\frac{g\tilde{h}l^Y}{(1-s_A)l^A} = \sigma g + \rho - n. \qquad (32)$$

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According to (3) and (14), we obtain

$$\frac{l^Y}{l^A} = \frac{1 - \int_{\tilde{h}}^{\bar{h}} f(h)\mathrm{d}h}{\int_{\tilde{h}}^{\bar{h}} h f(h)\mathrm{d}h}. \tag{33}$$

Substituting (33) into (32) leads to

$$\Gamma(\tilde{h}) \equiv \frac{\left(1 - \int_{\tilde{h}}^{\bar{h}} f(h)\mathrm{d}h\right)\tilde{h}}{\int_{\tilde{h}}^{\bar{h}} h f(h)\mathrm{d}h} = (1 - s_A)\frac{\frac{1}{\alpha} - 1}{1 - \frac{1}{\kappa}}\frac{\sigma g + \rho - n}{g} \equiv \Omega(s_A). \tag{34}$$

As $\Gamma'(\tilde{h}) > 0$, the solution of (34) for $\tilde{h}^*$ is unique. Moreover, noting that $\Omega'(s_A) < 0$ and the fact that s_K does not enter (34) confirms comparative-static results in part (ii) of Proposition 1.

That Y and K growth with rate $n + g$ in the long run follows from (10) and (11). Finally, we show that the transversality condition (23) holds under assumption (A1). First, use (21) and $\dot{c}(j)/c(j) = g$ to find that $\dot{\lambda}(j)/\lambda(j) = -\sigma g$. It remains to be shown that $a(j)$ grows with rate g in the long run. Rewriting (6) to

$$\frac{\dot{a}(j)}{a(j)} = r - n + \frac{I(j)}{a(j)} - \frac{c(j)}{a(j)} - \frac{T}{a(j)} \tag{35}$$

reveals that $\dot{a}(j)/a(j) = g$ indeed holds in steady state, if both the lump-sum tax per household (T) and income level $I(j)$ grow at rate g. Recalling from (29) that the long run interest rate r is time-invariant, we see from (27) that wages rates w^A and $w^Y = w^A\tilde{h}$, and therefore all income levels, grow in steady state at the same rate as A (namely at rate g). Moreover, defining $k \equiv K/N$, the lump-sum tax reads

$$T = s_K(r + \delta)k + s_A w^A l^A. \tag{36}$$

Use (36) together with the facts that w^A and k grow at rate g in the long run to see that $\dot{T}/T = g$ holds in steady state. This concludes the proof. ∎

Proof of Corollary 1: Part (i) is an immediate implication of the result that $\tilde{h}^*$ is decreasing in s_A (part (ii) of Proposition 1). To illustrate part (ii), suppose that the

distribution of R&D skills is uniform, i.e., $f(h) = (\bar{h} - \underline{h})^{-1}$. Thus, (34) becomes

$$\frac{(\tilde{h} - \underline{h})\tilde{h}}{\bar{h}^2 - \tilde{h}^2} = \Omega(s_A). \tag{37}$$

Rewriting (37) reveals that there is only one positive root for the steady state value of threshold skill level $\tilde{h}$, which is given by

$$\tilde{h}^* = \frac{\underline{h}}{2(1+\Omega)} \left(1 + \sqrt{1 + 4\Omega(s_A)\left[1 + \Omega(s_A)\right]\left(\frac{\bar{h}}{\underline{h}}\right)^2} \right). \tag{38}$$

Hence, the fraction of labor in BGE allocated to R&D, $d^* \equiv \int_{\tilde{h}^*}^{\bar{h}} f(h)\mathrm{d}h$, reads

$$d^* = \frac{\bar{h} - \tilde{h}^*}{\bar{h} - \underline{h}} = \frac{\frac{\bar{h}}{\underline{h}} - \frac{1}{2[1+\Omega(s_A)]}\left(1 + \sqrt{1 + 4\Omega(s_A)\left[1 + \Omega(s_A)\right]\left(\frac{\bar{h}}{\underline{h}}\right)^2} \right)}{\frac{\bar{h}}{\underline{h}} - 1}. \tag{39}$$

This reveals that, for a uniform skill distribution, d^* can be written as function of $\bar{h}/\underline{h}$. However, the relationship is ambiguous. ∎

Proof of Proposition 2: The current-value Hamiltonian which corresponds to the social planning problem (16) is given by

$$\mathcal{H} = \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda_k [\overbrace{A^{1-\alpha}k^\alpha \underbrace{\left(1 - \int_{\tilde{h}}^{\bar{h}} f(h)\mathrm{d}h \right)}_{=l^Y} - (\delta + n)k - c}^{=\dot{k}}] + \lambda_A \overbrace{\nu A^\phi N \underbrace{\int_{\tilde{h}}^{\bar{h}} h f(h)\mathrm{d}h}_{=l^A}}^{=\dot{A}}, \tag{40}$$

where λ_k and λ_A are co-state variables associated with constraints (15) and (2), respectively. Necessary optimality conditions are $\partial\mathcal{H}/\partial c = \partial\mathcal{H}/\partial\tilde{h} = 0$ (control variables), $\dot{\lambda}_z = (\rho - n)\lambda_z - \partial\mathcal{H}/\partial z$ for $z \in \{k, A\}$ (state variables), and the corresponding transversality conditions. Thus,

$$\lambda_k = c^{-\sigma}, \text{ i.e., } \frac{\dot{\lambda}_k}{\lambda_k} = -\sigma\frac{\dot{c}}{c}, \tag{41}$$

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$$(1 - \alpha) \left(\frac{k}{Al^Y} \right)^\alpha = \frac{\lambda_A}{\lambda_k} \nu A^{\phi-1} N \tilde{h}, \tag{42}$$

$$\frac{\dot{\lambda}_k}{\lambda_k} = \rho - \alpha \underbrace{\left(\frac{Al^Y}{k} \right)^{1-\alpha}}_{=Y/K} + \delta, \tag{43}$$

$$\frac{\dot{\lambda}_A}{\lambda_A} = \rho - n - \frac{\lambda_k}{\lambda_A} (1 - \alpha) \left(\frac{k}{A} \right)^\alpha (l^Y)^{1-\alpha} - \phi \frac{\dot{A}}{A}, \tag{44}$$

$$\lim_{t \to \infty} \lambda_{z,t} e^{-(\rho-n)t} z_t = 0, \ z \in \{k, A\}. \tag{45}$$

($\lambda_{z,t}$ denotes the co-state variable associated with state variable z at time t.)

Solving (42) for λ_k/λ_A and using (31) implies

$$\frac{\lambda_k}{\lambda_A} = \frac{g\tilde{h}}{(1 - \alpha)l^A} \left(\frac{Al^Y}{k} \right)^\alpha. \tag{46}$$

Suppose that A and k grow ar rate n in the long run, whereas $\tilde{h}$ and thus also l^Y and l^A are stationary. These properties are easily confirmed, as are transversality conditions. From (46), we then find $\dot{\lambda}_k/\lambda_k = \dot{\lambda}_A/\lambda_A$. Moreover, using $\dot{c}/c = g$ in (41) implies that

$$\frac{\dot{\lambda}_k}{\lambda_k} = \frac{\dot{\lambda}_A}{\lambda_A} = -\sigma g. \tag{47}$$

Using $\dot{\lambda}_A/\lambda_A = -\sigma g$, $\dot{A}/A = g = \frac{n}{1-\phi}$ and (46) in (44), we obtain

$$\tilde{h} \frac{l^Y}{l^A} = \frac{\rho + (\sigma - 1)g}{g}. \tag{48}$$

Using (33) and the definition of function $\Gamma(\cdot)$, we find that the optimal threshold skill level, $\tilde{h}^{opt}$, is uniquely given by

$$\Gamma(\tilde{h}^{opt}) = \frac{\rho + (\sigma - 1)g}{g}. \tag{49}$$

Recall that $\Gamma' > 0$. Thus, according to (34) and (49), for $s_A = 0$ we find that

$\tilde{h}^* > \tilde{h}^{opt}$ if and only if

$$\frac{\frac{1}{\alpha} - 1}{1 - \frac{1}{\kappa}}(\sigma g + \rho - n) > \rho + (\sigma - 1)g.$$

Since $1 < \kappa < 1/\alpha$, we have $1/\alpha - 1 > 1 - 1/\kappa$. Moreover, using $g = \frac{n}{1-\phi}$, we find that $\sigma g + \rho - n \geq \rho + (\sigma - 1)g$ holds if and only if $\phi \geq 0$. This confirms that $\tilde{h}^* > \tilde{h}^{opt}$ if $s_A = 0$ and $\phi \geq 0$. Finally, use (10) and (11) to see that Y and K growth with rate $n + g$ in the long run. This concludes the proof. ∎

Proof of Proposition 3: First, substitute (29) into (11) to see that the capital-output ratio in decentralized BGE is given by

$$\left(\frac{K}{Y}\right)^* = \frac{\alpha}{\kappa(1 - s_K)(\sigma g + \rho + \delta)}. \tag{50}$$

Second, substitute $\dot{\lambda}_K/\lambda_K = -\sigma g$ from (47) into (43) to find

$$\alpha \left(\frac{Al^Y}{k}\right)^{1-\alpha} = \sigma g + \rho + \delta. \tag{51}$$

Since $\left(Al^Y/k\right)^{1-\alpha} = Y/K$, the socially optimal capital-output ratio is given by

$$\left(\frac{K}{Y}\right)^{opt} = \frac{\alpha}{\sigma g + \rho + \delta}. \tag{52}$$

The optimal capital subsidy, s_K^{opt}, follows from setting $(K/Y)^* = (K/Y)^{opt}$. To find the optimal R&D subsidy, s_A^{opt}, set the right-hand sides of (34) and (49) equal to each other. ∎